

AGU Ocean Sciences 2024 - Additional Info for Poster AI24B-2373

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1 Technical Description

The spatial scales are estimated by fitting ellipse-shaped (major and minor axes plus rotation angle; L_x, L_y, θ , Equation 3 below) non-stationary anisotropic covariance function to the observed anomaly covariances [Paciorek and Schervish, 2006, Karspeck et al., 2012]. The chosen function is the Matérn covariance, which allows a flexible shape (ν) between the exponential ($\nu = 0.5$) and Gaussian ($\nu \rightarrow \infty$) limit. ν is usually fixed and is set to the 0.5 exponential limit.

Under this model, the covariance between points x and x' is:

$$c(x, x') = \frac{\sigma\sigma'}{\Gamma(\nu)2^{\nu-1}} \frac{|\Sigma|^{1/4} |\Sigma'|^{1/4}}{|\bar{\Sigma}|^{1/2}} (2\bar{\tau}\sqrt{\nu})^\nu K_\nu(2\bar{\tau}\sqrt{\nu}) \quad (1)$$

Γ is the gamma function, K_ν is the modified Bessel function of the 2nd kind, and $||$ indicates the determinant. σ and σ' are the standard deviations at x and x' . $\bar{\tau}$ is the normalised (Mahalanobis) distance between the 2 points with the normalisation Σ being the anisotropic length scales (L_x (major axis), L_y (minor axis), and θ (rotation angle).

$$\bar{\tau} = \sqrt{(x - x')^T \Sigma^{-1} (x - x')} \quad (2)$$

$$\Sigma = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} L_x & 0 \\ 0 & L_y \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}^T \quad (3)$$

$$\bar{\Sigma} = \frac{\Sigma + \Sigma'}{2} \quad (4)$$

L_x, L_y, θ are determined numerically with maximum likelihood estimation, using a moving window, capturing their spatial non-stationarity. For this estimation, each moving window is fitted with the stationary form of Equation 1 (i.e. without the middle Σ fraction). We seek the best parameters to Equation

4 (enclosed in τ within Equation 1) that minimises the (negative) log-likelihood loss function.

The stationary estimates can then be stitched together using Equation 1, creating non-stationary covariance and correlation matrices. The Σ fraction essentially averages the length scales and rotation angles between different locations.

2 Useful papers

Paciorek and Schervish 2006:



<https://onlinelibrary.wiley.com/doi/10.1002/env.785>

Karspeck et al 2012:



<https://onlinelibrary.wiley.com/doi/10.1002/qj.900>

References

Christopher J. Paciorek and Mark J. Schervish. Spatial modelling using a new class of nonstationary covariance functions. *Environmetrics*, 17(5):483–506, August 2006. ISSN 1180-4009, 1099-095X. doi: 10.1002/env.785. URL <https://onlinelibrary.wiley.com/doi/10.1002/env.785>.

Alicia R. Karspeck, Alexey Kaplan, and Stephan R. Sain. Bayesian modelling and ensemble reconstruction of mid-scale spatial variability in North Atlantic sea-surface temperatures for 1850-2008. *Quarterly Journal of the Royal Meteorological Society*, 138(662):234–248,

January 2012. ISSN 00359009. doi: 10.1002/qj.900. URL <https://onlinelibrary.wiley.com/doi/10.1002/qj.900>.